Aeroacoustics & Noise Control Lab.

Dept. of Mechanical and Aerospace Engineering

Noise Engineering & Aeroacoustics **Class 2019_Spring**

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TERMPROJECT#1 (Due 05/22)

1. Obtain a solution for the following 1-D wave equation using the numerical scheme (see attached appendix) and compare to the analytic solution (also, derive the analytic solution): Draw the graph of 'u' as a function of 'x' with various conditions given at the below. Discuss the results.

Equation: $\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = 0$

Initial condition: $\mathrm{u(x)} = 0.1\exp\left(-ln2 \times \left(\frac{x}{5}\right)^2\right)$

2. Obtain a solution for the following 3-D wave equation the using numerical scheme (see attached appendix) and compare to the analytic solution (also, derive the analytic solution): Draw the graph of 'u' as a function of 'r' with various conditions given at the below. Discuss the results.

Equation: $\frac{\partial u}{\partial t} + \frac{u}{r} + \frac{\partial u}{\partial r} = 0, \quad \mathrm{r} > 5$

Boundary condition: $r = 5, \quad u = \sin(\omega t), \quad t > 0, \quad \omega = 0.25\pi$

Appendix

※ Numerical analysis condition

Calculation area: $-50 < x < 200$ (problem 1)

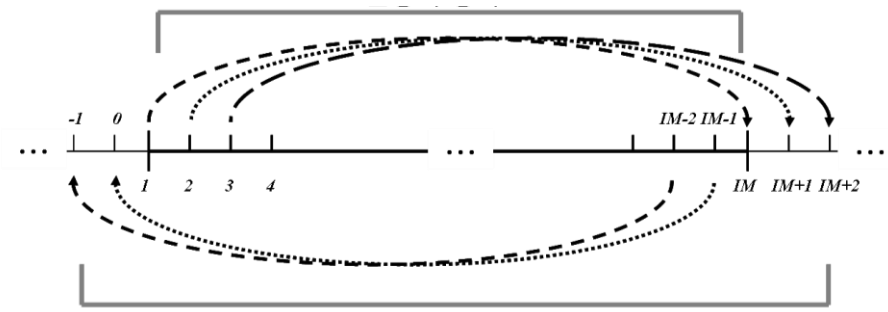
$5 < r < 200$ (problem 2)

Time to show the results: $t = 20s, 40s, 60s, 80s, 100s$

Grid size: $\Delta x, \Delta r = 0.5 \;/\; \Delta t = 0.05$

Boundary condition:

Problem 1: periodic boundary condition



U(1) = U(IM)

U(2) = U(IM+1)

U(3) = U(IM+2)

U(4) = U(IM+3)

U(0) = U(IM-1)

U(-1) = U(IM-2)

U(-2) = U(IM-3)

Problem 2:

Inlet B.C (U(0), U(-1), U(-2)) : Use the exact solution at the location and time

Outlet B.C (U(IM+1), U(IM+2), U(IM+3)): Use linear interpolation

U(IM+1) = 2U(IM) – U(IM-1)

U(IM+2) = 2U(IM+1) – U(IM)

U(IM+3) = 2U(IM+2) – U(IM+1)

※ Numerical scheme

Space differentiation: Dispersion-Relation-Preserving

$$\left(\frac{\partial f}{\partial x}\right)_l = \frac{1}{\Delta x}\sum_{j=-3}^{3} a_j f\left(x + j\Delta x\right); \qquad a_{-j} = -a_j$$

$a_0 =$ 0.0

$a_1 = -a_{-1} =$ 0.770882380518

$a_2 = -a_{-2} = -0.166705904415$

$a_3 = -a_{-3} =$ 0.0208431427703

Time differentiation: 4th order Optimized Adams-Bashforth method

$k_1 = f\left(x_n, t - 3\Delta t\right)$

$k_2 = f\left(x_n, t - 2\Delta t\right)$

$k_3 = f\left(x_n, t - \Delta t\right)$

$k_4 = f\left(x_n, t\right) \qquad if\ t < 0 \quad then\ f(t) = 0$

$y^{n+1} = y^n + \Delta t\left\{B_0 k_4 + B_1 k_3 + B_2 k_2 + B_3 k_1\right\}$

B0=2.302558088838

B1=-2.491007599848

B2=1.574340933182

B3=-0.385891422172